Student Workbook

Conic Curves in Polar Form

Name: ________________________ Date: ______________ Class: ________

ESSENTIAL QUESTION

How does the unified polar form $r = ed/(1 \pm e\cos\theta)$ or $r = ed/(1 \pm e\sin\theta)$ describe all conic sections, and how does eccentricity e determine which type of conic we get?

Lesson Overview

Placing the focus of a conic at the pole unifies all three conic types into one family of equations. The eccentricity e alone determines the shape — no separate formulas for ellipse, parabola, and hyperbola are needed — and the sign pattern tells you where the directrix sits relative to the pole.

$$r = ed/(1 + e\cos\theta)$$
$$r = ed/(1 - e\cos\theta)$$
$$r = ed/(1 + e\sin\theta)$$
$$r = ed/(1 - e\sin\theta)$$

Directrix Right	$r = ed/(1+e\cos\theta)$; directrix $x=d$ (right of pole); conic opens left.
Directrix Left	$r = ed/(1-e\cos\theta)$; directrix $x=-d$ (left of pole); conic opens right.
Directrix Above	$r = ed/(1+e\sin\theta)$; directrix $y=d$ (above pole); conic opens down.
Directrix Below	$r = ed/(1-e\sin\theta)$; directrix $y=-d$ (below pole); conic opens up.
Eccentricity	$0<e<1$ → Ellipse. $e=1$ → Parabola. $e>1$ → Hyperbola. d = focus-to-directrix distance.

Worked Examples

EXAMPLE 1

Identify the conic and find key features: $r = 6/(1 + 2\cos\theta)$.

- Form: $r = ed/(1+e\cos\theta)$; $e = 2$, $ed = 6 \rightarrow d = 3$
- $e = 2 > 1$ → Hyperbola; directrix: $x = 3$ (right of pole)
- $\theta=0$: $r=6/3=2$; $\theta=\pi$: $r=6/(1-2)=-6$ (magnitude 6, opposite direction)

Answer: Hyperbola; $e=2$, $d=3$; vertices at $r=2$ ($\theta=0$) and $r=6$ ($\theta=\pi$)

EXAMPLE 2

Identify the conic: $r = 4/(1 - \sin\theta)$.

- Form: $r = ed/(1-e\sin\theta)$; $e = 1$, $ed = 4 \rightarrow d = 4$
- $e = 1 \rightarrow$ Parabola; directrix: $y = -4$ (below pole)
- Vertex: $\theta=3\pi/2$: $r=4/(1-(-1))=4/2=2$

Answer: Parabola; $e=1$, vertex at $r=2$ when $\theta=3\pi/2$

EXAMPLE 3

Write the polar equation of a parabola with focus at origin and directrix $x = -3$.

- Directrix left of pole $\rightarrow$ form $r = ed/(1-e\cos\theta)$
- $e = 1$ (parabola), $d = 3$
- $r = (1)(3)/(1-\cos\theta) = 3/(1-\cos\theta)$

Answer: $r = 3/(1-\cos\theta)$

EXAMPLE 4

Identify and find vertices: $r = 10/(5 + 4\cos\theta)$.

- Divide by 5: $r = 2/(1 + (4/5)\cos\theta)$
- $e = 4/5 < 1 \rightarrow$ Ellipse; $ed = 2 \rightarrow d = 2/(4/5) = 5/2$
- Vertices: $\theta=0$: $r=10/9$; $\theta=\pi$: $r=10/1=10$

Answer: Ellipse; $e=4/5$; vertices $r=10/9$ ($\theta=0$) and $r=10$ ($\theta=\pi$)

EXAMPLE 5

Convert $r = 8/(2 + 2\sin\theta)$ to rectangular form.

- Divide by 2: $r = 4/(1+\sin\theta)$; $e=1 \rightarrow$ parabola
- $r(1+\sin\theta) = 4 \rightarrow r + r\sin\theta = 4 \rightarrow r = 4 - y$ (since $r\sin\theta = y$)
- $\sqrt{(x^2+y^2)} = 4 - y \rightarrow x^2+y^2 = 16 - 8y + y^2 \rightarrow x^2 = 16 - 8y$

Answer: $x^2 = -8(y-2)$; parabola opening downward, vertex $(0,2)$

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Identify the conic and find e and d: $r = 12/(3 + 3\cos\theta)$.

Hint: Divide numerator and denominator by 3 first to get the standard form $r = ed/(1+e\cos\theta)$.

2 Identify the conic: $r = 15/(3 - 5\sin\theta)$.

Hint: Divide by 3 to get standard form. Compare e to 1.

3 Write the polar equation of an ellipse with $e = 1/2$ and directrix $y = 6$.

Hint: Directrix above pole → form $r = ed/(1+e\sin\theta)$. Substitute $e=1/2$, $d=6$.

4 Find the vertices of $r = 9/(3 + 6\sin\theta)$ by substituting $\theta = \pi/2$ and $\theta = 3\pi/2$.

Hint: Divide by 3 first: $r = 3/(1+2\sin\theta)$. Then substitute.

5 Convert $r = 6/(1 + \cos\theta)$ to rectangular form.

Hint: Multiply both sides by $(1+\cos\theta)$: $r + r\cos\theta = 6$. Use $r\cos\theta = x$ and $r = \sqrt{x^2+y^2}$.

Key Vocabulary

Polar form of a conic	$r = ed/(1\pm e\cos\theta)$ or $r = ed/(1\pm e\sin\theta)$; focus at the pole.
Pole	The origin of the polar coordinate system; coincides with the focus of the conic.
Eccentricity (e)	Determines conic type: $0<e<1$ ellipse, $e=1$ parabola, $e>1$ hyperbola.
Directrix	The fixed line used in the focus-directrix definition; distance d from the pole.
Vertex (polar)	Found by substituting $\theta = 0, \pi, \pi/2$, or $3\pi/2$ into the polar equation.
Unified polar form	A single equation form that represents all three conic types based on e.

Quick Check Quiz

Circle the letter of the correct answer for each question.

1 **For $r = 5/(1 + \cos\theta)$, the conic is a:** Multiple Choice

A Ellipse
B Parabola
C Hyperbola
D Circle

2 **For $r = 12/(3 + 4\cos\theta)$, after dividing by 3, e =** Multiple Choice

A 3
B 4
C 4/3
D 3/4

3 **$r = ed/(1 - e\sin\theta)$ has its directrix:** Multiple Choice

- **A** Above the pole
- **B** Below the pole
- **C** Right of the pole
- **D** Left of the pole

4 **An ellipse in polar form has e =** Multiple Choice

- **A** Equal to 1
- **B** Greater than 1
- **C** Between 0 and 1
- **D** Equal to 0

5 **For $r = 6/(2 + \sin\theta)$, dividing by 2 gives e =** Multiple Choice

- **A** 2
- **B** 1/2
- **C** 6
- **D** 3

Common Mistakes to Avoid

✗ Forgetting to divide by the constant to get standard form before identifying e.

✓ Always rewrite as $r = ed/(1 \pm e\cos\theta)$ with coefficient 1 in the denominator. Divide numerator and denominator by the constant term.

✗ Confusing which form ($\cos\theta$ vs $\sin\theta$) corresponds to which directrix orientation.

✓ $\cos\theta \to$ vertical directrix (left/right of pole); $\sin\theta \to$ horizontal directrix (above/below pole).

✗ Thinking e is the coefficient of $\cos\theta$ or $\sin\theta$ before dividing.

✓ e is the coefficient AFTER dividing to get standard form. In $r=12/(3+4\cos\theta)$, $e=4/3$ (not 4).

✗ Using $\theta=0$ and $\theta=\pi/2$ to find vertices for a $\sin\theta$ form.

✓ For $\cos\theta$ forms, use $\theta=0$ and $\theta=\pi$ for vertices. For $\sin\theta$ forms, use $\theta=\pi/2$ and $\theta=3\pi/2$.

Math Tips

★ Always divide first: get the denominator to start with 1 before reading off e and d.

★ Eccentricity memory: $e<1$ Ellipse, $e=1$ Parabola, $e>1$ Hyperbola. Think 'EPH' — Ellipse, Parabola, Hyperbola.

★ The sign in the denominator tells you the directrix location: $+\cos\theta$ → directrix right; $-\cos\theta$ → directrix left; $+\sin\theta$ → directrix above; $-\sin\theta$ → directrix below.

★ To convert to rectangular: use $r^2=x^2+y^2$, $r\cos\theta=x$, $r\sin\theta=y$, and $r=\sqrt{(x^2+y^2)}$.

★ Vertices are the closest and farthest points from the focus. For $r=ed/(1+e\cos\theta)$: closest at $\theta=0$, farthest at $\theta=\pi$.